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Algebraic Number Theory

Exercise sheet 2

Solutions should be submitted online before 04.05.20 via the Moodle page of the course:
`https://moodle.hu-berlin.de/course/view.php?id=95156`

Exercise 2.1. (3 points) Let $K \subset L$ be a Galois extension and $f \in K[X]$ an irreducible polynomial of degree n that splits in $L[X]$. Let $\{x_1, \ldots, x_n\} \subset L$ be the roots of f. Prove that $\mathrm{Gal}(L/K)$ acts transitively on $\{x_1, \ldots, x_n\}$.

Exercise 2.2. (3 points) Let K be a field, $\mathrm{char}(K) = 0$, and $f \in K[X]$ an arbitrary polynomial. Let $L \subset \overline{K}$ be the subfield generated by all the roots of f in $\overline{K}$. Prove that $K \subset L$ is a Galois extension (L is called the splitting field of f). Prove that $[L : K] \leqslant \deg(f)!$

Exercise 2.3. (3 + 3 points) Describe the ring of integers and determine the signature of the trace form for the following fields:

1. $\mathbb{Q}(\sqrt{7})$
2. $\mathbb{Q}(\sqrt{-11})$

Exercise 2.4. (3 points) Prove that the ring $\mathbb{Z}[\sqrt{-3}]$ is not integrally closed in its field of fractions, compute its integral closure.