'Hyperbolicity of complex varieties'

Winter term 2016/17, Tuesday 2-4pm, 0.011

The aim of the seminar is to understand the ideas behind recent works on Kobayashi hyperbolicity and Green-Griffiths-Lang conjecture. We will start from the classical material on the relations between different notions of hyperbolicity for complex varieties, discuss basic examples and constructions. Then we will study more advanced techniques developed by Green-Griffiths, Demailly, McQuillan, Siu and others, discuss applications of these techniques to hyperbolicity of surfaces.

For further information or if you want to give a talk in the seminar, please contact huybrech@ or aosoldat@. The **first talk will be on October 25.**

25 October: Introduction (Speaker: Andrey Soldatenkov)
A general introduction to the topic of hyperbolicity of complex varieties will be given, following mostly the survey paper of Lang [25] and introductory chapters of Demailly's [13]. We will discuss various notions of hyperbolicity and relations between them, prove some basic properties. The talk is expected to be elementary.

15 November: End of the introduction (Speaker: Andrey Soldatenkov)
Varieties with ample cotangent bundle (Speaker: Emma Brakkee)
The talk should follow the exposition in Debarre's notes [7]. After presenting some elementary preliminary material (the notion of ampleness for vector bundles, Ahlfors-Schwarz lemma), it should be proven that varieties with ample cotangent bundle are hyperbolic. After that various constructions of varieties with ample cotangent bundle should be discussed, following the same notes by Debarre and his paper [8].

22 November: Varieties with ample cotangent bundle II (Speaker: Emma Brakkee)

29 November: Introduction to jet differentials (Speaker: Francesco Genovese)
Jet differentials are the technical heart of modern approaches to hyperbolicity, so we should spend some time studying them. The idea of using jets goes back to A. Bloch. We should start to discuss Part A of the Green-Griffiths paper [23] (skipping the piece about surfaces of general type). The following things should be explained: jet bundles, jet differentials, k-jet metrics with negative curvature. One can follow the more detailed exposition in [11], §§6–8. Some parts can be postponed until the next talk.

13 December: Jet differentials and Bloch's theorem (Speaker: Daniel Huybrechts)
This should be a continuation of the previous talk. After finishing the introduction to jet differentials, the proof of A. Bloch's theorem should be explained. One can follow either [23] or [11], §10. Other references include [18] and [27].

10 January: Curves on surfaces of general type (Speaker: Vladimir Lazić)
Bogomolov has proven in [2] that on a surface of general type with $c_1^2 > c_2$ curves of any fixed geometric genus form a bounded family. An overview of this and related results is given in [3] and [9]. We can also follow section 7 in [7]. The proof uses some foliation theory. Then we can discuss the theorem of Lu-Yau about entire curves on surfaces of general type with $c_1^2 > 2c_2$ which can also be found in [7].

17 January: Algebraic hyperbolicity of hypersurfaces in projective space (Speaker: Corinne Bedussa)
Clemens has shown in [6] that a general hypersurface in $\mathbb{P}^n$ of degree $\geqslant 2n - 1$ does not contain rational curves. His results have been subsequently improved by Ein [20], [21], Voisin [34] and Pacienza [30]. In fact, it follows from these works that a general hypersurface in $\mathbb{P}^n$ of sufficiently high degree is algebraically hyperbolic. The aim of the talk is to explain this result. One can follow one of the mentioned papers or section 6 of [7], the notes [35] can also be useful. The proofs here are purely algebraic and do not use any analytic techniques.

24 January: Complex surfaces dominated by $\mathbb{C}^2$ (Speaker: Stefan Schreieder)
The talk should give an overview of results of Buzzard and Lu [4]. It would be especially interesting to understand examples of K3 surfaces dominated by $\mathbb{C}^2$ (section 4 in [4]). The paper [32] may be helpful.

31 January: Hyperbolic surfaces in $\mathbb{P}^3$ (Speaker: Ulrike Rieß)
It is conjectured that a generic surface in $\mathbb{P}^3$ of degree $\geqslant 5$ is hyperbolic. Weaker bounds have been obtained by Demailly-El Goul [10] (degree $\geqslant 21$) and Paun [31] (degree $\geqslant 18$). The talk should give a survey of the paper [10], with the ideas of the proofs if possible. An independent approach has been developed by McQuillan [28]. Other references are [31] and [18]. Another possible topic is the construction of explicit examples of hyperbolic surfaces of low degree: see [19] (degree 6) or section 5 in [7] (degree 8). See also [29], [22].

7 February: (The topic is open for discussion) (Speaker: ?)

References

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