Prof. Dr. Daniel Huybrechts
Dr. Andrey Soldatenkov Winter term 2015/16

Exercises, Algebraic Geometry I – Week 14

Exercise 75. *Rational functions with prescribed poles.*
Let X be an integral, regular curve, projective over an algebraically closed field k. Prove the following assertions.

i) For every closed point $x \in X$ there exists a rational function $f \in K(X)$ with a pole at x (of some order $n > 0$) and no pole anywhere else.

ii) For given closed points $x_1, \ldots, x_r \in X$ there exists a rational function $f \in K(X)$ with poles at all x_i (of some order $n_i > 0$) and no pole anywhere else.

Exercise 76. *Gonality of a curve.*
Under the assumptions of the previous exercise, show that there exists a finite morphism $f\colon X \to \mathbb{P}^1_k$ of degree $\leq g(X) + 1$. In other words, the gonality of a curve X is bounded by $\mathrm{gon}(X) \leq g(X) + 1$.

Exercise 77. *Inflection points of plane cubics.*
Let $X \subset \mathbb{P}^2_k$, $k = \bar{k}$ be a regular cubic with an inflection point $x_0 \in X$ leading to a group structure on the set of closed points of X with x_0 as the origin. Show that any other inflection point $x \in X$ is of order 3. Is the converse also true?

Exercise 78. *Base locus of linear systems.*
The base locus of a linear system $\mathbb{P}(V) \subset |\mathcal{L}|$ is by definition the set $\mathrm{Bs}(V) := \bigcap_{s \in V} (s)_0$.

i) Show that (under appropriate conditions on X) $\mathrm{Bs}(V)$ is a closed subset.

ii) Determine the base locus of the *Hesse pencil* $V := \{f_{t_0,t_1} := t_0(x_0^3 + x_1^3 + x_2^3) + t_1 x_0 x_1 x_2\} \subset H^0(\mathbb{P}^2_k, \mathcal{O}(3))$, assuming that $k = \bar{k}$. Show that they are all inflection points of all X_λ defined by $f_{1,\lambda}$.

Exercise 79. *Projection from a linear subspace.*
Describe the linear system $V \subset H^0(\mathbb{P}^n, \mathcal{O}(1))$ that yields the linear projection from the plane $\mathbb{P}^2 \subset \mathbb{P}^n$ defined by $x_4 = \ldots = x_{n+1} = 0$ onto $\mathbb{P}^{n-3} \subset \mathbb{P}^n$ defined by $x_0 = x_1 = x_2 = 0$.

Exercise 80. *Pull back of divisors.*
Let $\varphi\colon X \to Y$ be a finite morphism of regular curves projective over a field $k = \bar{k}$. Show that $\varphi^*\mathcal{O}(D) \cong \mathcal{O}(\varphi^* D)$ for all divisors D on Y.

These exercises should be discussed in the tutorials.
No lecture on Monday February 8. Last lecture on Thursday February 11. All the material, including this sheet, is relevant for the exam.

Recommendation for the summer term

Lecture courses

Algebraic Geometry II. (V4A2) D. Huybrechts, A. Soldatenkov, Monday 4-6pm, Thursday 2-4 pm (GHS)

Selected Topics in Algebraic Geometry: Mixed Hodge structures and geometry (V5A4) St. Schreieder, Wednesday 12am-2pm, (SR 0.006)
`http://www.math.uni-bonn.de/people/schreied/Hodge-Theory-SS16.pdf`

Seminars

Abelian varieties (S4A1) V. Lazic, L. Tasin, Tuesday 12am-2pm (SR 1.007)
`http://www.math.uni-bonn.de/people/lazic/S4A1-2016.pdf`
Vorbesprechung/preliminary meeting: February 2 **!!!!!!**, 11am (Hausdorff Raum)

Stable reduction of curves (S4A3) E. Hellmann, M. Rapoport, Tue 4-6pm (SR 0.006)
`http://www.math.uni-bonn.de/people/hellmann/reductionofcurves/programm.pdf`