Prof. Dr. Daniel Huybrechts
Dr. Andrey Soldatenkov

Summer term 2016

Exercises, Algebraic Geometry II – Week 1

Exercise 1. (6 points) *Sober topological spaces.*
A topological space X is *sober* if every closed irreducible set $Y \subset X$ contains a unique generic point.

1. Show that the topological space underlying an arbitrary scheme is sober.
2. Show that for any topological space X the space $t(X)$ of all closed irreducible sets with the topology defined in class is sober.
3. Let $(sobTop) \subset (Top)$ be the full subcategory of sober topological spaces of the category of all topological spaces. Consider $X \mapsto t(X)$ as a functor $t\colon (Top) \to (sobTop)$. Show that it is left adjoint to the inclusion.

Exercise 2. (4 points) *Maximal open sets of definition of rational functions and rational maps.* Consider varieties X and Y (over an algebraically closed field k).

1. Show that for every rational function $f \in K(X)$ there exists a maximal open subset on which f is regular.
2. Show that for every rational map $f\colon X \dashrightarrow Y$ there exists a maximal open subset on which f is a morphism.
3. Suppose f in 2) is a birational map. Is then the restriction $f|_U$ to the maximal open subset on which f is regular always injective?

Exercise 3. (3 points) *Cremona transformations.*
Solve Exercise I.4.6 in Hartshorne's book.

Exercise 4. (2 points) *Dominant rational maps.*
Are there any dominant rational maps $\mathbb{P}^2 \dashrightarrow \mathbb{P}^1$?

Exercise 5. (2 points) *Blow-up.*
Solve Exercise I.4.10 in Hartshorne's book.

Due Monday 18 April, 2016. Before(!!) the lecture.