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Summer term 2016

Preparing the exam: basic questions

1. Find examples of morphisms which are smooth but not unramified; unramified but not étale; flat but not smooth.
2. Let $f\colon X \to Y$ be a smooth projective morphism, Y Noetherian scheme. Consider the Stein factorization $f = g \circ h$, $h\colon X \to Y'$, $g\colon Y' \to Y$. Is g étale?
3. Let $A = k[x,y,z]/(xy, yz, zx)$, $\mathfrak{m} = (x,y,z)$, and $B = k[x,y]/(xy(x-y))$, $\mathfrak{n} = (x,y)$. Is it true that $\hat{A}_{\mathfrak{m}} = \hat{B}_{\mathfrak{n}}$?
4. Let $C \simeq \mathbb{P}^1 \hookrightarrow \mathbb{P}^3$ be a line. Then $T_{\mathbb{P}^3}|_C \simeq \mathcal{O}(a) \oplus \mathcal{O}(b) \oplus \mathcal{O}(c)$. Find a, b, c.
5. Find an example of a surjective proper morphism $f\colon X \to Y$ such that $f_*\mathcal{O}_X$ has non-zero torsion.
6. Let X, Y, Z be integral schemes, $f\colon X \to Y$, $g\colon Y \to Z$ two morphisms, f and $g \circ f$ flat. Is g also flat?
7. Find an example of a non-smooth morphism between varieties with all fibres smooth.
8. Consider the rational map $\mathbb{P}^2 \dashrightarrow \mathbb{P}^2$, $[x:y:z] \mapsto [yz:xz:xy]$. Find the maximal open subset where this map is defined. Describe the exceptional locus.
9. Let A be a Noetherian ring and $\mathfrak{m} \subset A$ a maximal ideal. Is the $\mathfrak{m}$-adic completion $\hat{A}$ a local ring?
10. Let $X = \mathrm{Spec}(k[x,y,z,w]/(xy - zw))$, $Y = \mathrm{Spec}(k[x,y,z,w]/(x,z)) \subset X$. What is the exceptional locus of the blow-up morphism $\mathrm{Bl}_Y X \to X$?
11. Let k be a field and $A = k[\varepsilon]/(\varepsilon^2)$ the "ring of dual numbers". A finite A-module is a finite-dimensional k-vector space V with an endomorphism $E \in \mathrm{End(V)}$, such that $E^2 = 0$. What are the conditions for (V, E) to be a flat A-module?
12. Does there exist a double covering of $\mathbb{P}^1$ ramified in 3 points?
13. What are the Hodge numbers of the hypersurface in $\mathbb{P}^3$ given by $xy - zw = 0$?
14. Let X be a smooth variety and $Y \subset X$ a smooth subvariety. For the blow-up $\pi\colon \tilde{X} = \mathrm{Bl}_Y X \to X$ compute $R^i\pi_*\mathcal{O}_{\tilde{X}}$ for $i \geq 0$.
15. Let E be a locally free sheaf on a scheme X and L an invertible sheaf. Is it true that $\mathbb{P}(E) \simeq \mathbb{P}(E \otimes L)$?
16. Let X be the hypersurface in $\mathbb{P}^3_k$ (k algebraically closed) given by $xy - zw = 0$. Let $p = [1:0:0:0] \in X$ and $\tilde{X} = \mathrm{Bl}_p X$. Let $\pi\colon \tilde{X} \to \mathbb{P}^2$ be the projection to a hyperplane in $\mathbb{P}^3$. Describe the fibres of π.
17. Given an integer $d \geq 2$ find an example of a normal two-dimensional local ring $(A, \mathfrak{m})$, such that $\dim_k(\mathfrak{m}/\mathfrak{m}^2) = d$, where $k = A/\mathfrak{m}$.
18. Let X_1, X_2 be two smooth projective varieties over a field k. Is it true that $\mathrm{Pic}(X_1 \times_k X_2) \simeq \mathrm{Pic}(X_1) \times \mathrm{Pic}(X_2)$?
19. Let C be a smooth curve of positive genus, E a locally free sheaf of rank two on C. Can there exist a dominant rational map $\mathbb{P}^n \dashrightarrow \mathbb{P}(E)$ for some n?
20. Find an example of a subvariety in $\mathbb{P}^n$ that is not a local complete intersection.
21. Find an example of two normal varieties X, Y and a morphism $f\colon X \to Y$ that is finite but not flat.

Exams:

- **02.08.2016, 9.00 - 11.00, Großer Hörsaal, Wegelerstr. 10;**
- **28.09.2016, 9.00 - 11.00, Großer Hörsaal, Wegelerstr. 10.**