Prof. Dr. Daniel Huybrechts
Dr. Andrey Soldatenkov

Winter term 2016/17

Intersection theory and pure motives, Exercises – Week 11

Exercise 48. *Grothendieck groups of subcategories.*
Find an explicit (geometric) example of a full abelian subcategory $\mathcal{A} \hookrightarrow \mathcal{B}$ of an abelian category $\mathcal{B}$ such that the induced map $K(\mathcal{A}) \to K(\mathcal{B})$ between their Grothendieck groups is not injective.

Exercise 49. *Grothendieck classes of singular varieties.*
Let C be an integral projective curve over a field k with $\mathrm{char}(k) = 0$. Assume that C has one singular point which is an ordinary double point. Describe the class $[C] \in K(\mathrm{Var}(k))$ as the image of an element in $\mathbb{Z}[\mathrm{SmProj}(k)]$ under the natural surjection

$$\mathbb{Z}[\mathrm{SmProj}(k)] \twoheadrightarrow \mathbb{Z}[\mathrm{SmProj}(k)]/(\text{blow-up}) \cong K(\mathrm{Var}(k)).$$

Exercise 50. *Grothendieck classes of curves.*
Let C, C' be two smooth projective curves over a field k with non-isogenous Jacobians.

(i) Show that then $\mathfrak{h}(C)$ and $\mathfrak{h}(C')$ are non-isomorphic.

(ii) Assume that homological and numerical equivalence coincide. Show that then (i) implies that $[C] \neq [C']$ in $K(\mathrm{Var}(k))$.

(iii) Show that at least for $\mathrm{char}(k) = 0$, the assertion in (ii) can also be deduced from the isomorphism $K(\mathrm{Var}(k))/(\mathbb{L}) \cong \mathbb{Z}[\mathrm{SB(k)}]$ (without assuming any of the standard conjectures).

Exercise 51. *Motives and Chow groups of powers of (elliptic) curves.*
(i) Let E be an elliptic curve over a field k. Describe the motive $\mathfrak{h}(S^n(E))$ of the symmetric product $S^n(E)$ of E in terms of $\mathfrak{h}(E)$.

(ii) What can be said about $\mathfrak{h}(S^n(C))$ of an arbitrary smooth projective curve C (with $C(k) \neq \emptyset$) and for $n > 2g(C) - 2$?

(iii) Recall from Exercise 28 that $\mathfrak{h}(S^n(C)) \cong \mathfrak{h}(C^n, p)$, where $p = (1/n) \sum_{\sigma \in \mathfrak{S}_n} [\sigma]$. How close does this get you to describing $\mathfrak{h}(C^n)$?

Due Wednesday 18 January, 2017.